

Assignments - Week 1

MATH 4510/5510 -SUMMER - May 31st, 2022

1 General problems

1. If 8 rooks (castles) are randomly placed on a chessboard, compute the probability that none of the rooks can capture any of the others. That is, compute the probability that no row or file contains more than one rook.

2. Two cards are randomly selected from an ordinary playing deck. What is the probability that they form a blackjack? That is, what is the probability that one of the cards is an ace and the other one is either a ten, a jack, a queen, or a king?

3. A small community organization consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children, and 1 has five children.
 - (a) If one of these families is chosen at random, what is the probability it has i children, $i = 1, 2, 3, 4, 5$?
 - (b) If one of the children is randomly chosen, what is the probability that child comes from a family having i children, $i = 1, 2, 3, 4, 5$?

4. A forest contains 20 elk, of which 5 are captured, tagged, and then released. A certain time later, 4 of the 20 elk are captured. What is the probability that 2 of these 4 have been tagged? What assumptions are you making?

5. Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that
 - (a) 3 red, 2 blue, and 2 green balls are withdrawn;
 - (b) at least 2 red balls are withdrawn;
 - (c) all withdrawn balls are the same color;
 - (d) either exactly 3 red balls or exactly 3 blue balls are withdrawn.

6. There are 5 hotels in a certain town. If 3 people check into hotels in a day, what is the probability that they each check into a different hotel? What assumptions are you making?

7. If N people, including A and B, are randomly arranged in a line, what is the probability that A and B are next to each other?

2 Mathematics in Real Life

Probability as a tool to understand bias

Professor Carolyn Chun applied for tenure at the Naval Academy in 2021 alongside three other women and 11 men. The committee was supposed to select 10 out of the 15 candidates. Ten men received tenure, while no women were successful.

1. Based only on the above statement, do you think this is a situation of gender bias? Explain.
2. Assuming all the candidates had the same chances to get the tenure, what is the probability that only men get the tenure?

Conscious Consumption

Each year, 39,000 tons of unwanted clothes are thought to be sent from rich countries to the Chile's Atacama desert, such is our addiction to 'fast fashion'.

When you tire of that your things, this is where it may end up. We throw away some 13 million items of clothing every week. And according to the sustainability charity WRAP, 70 per cent of our used clothing is sent overseas, making the UK the world's second largest exporter of used clothing after the U.S.

Source:

<https://www.dailymail.co.uk/news/article-10450221/Dumped-Atacama-desert-mountain-d.html>

Question. Based on your reading of the above article, can you come up with a mathematical argument using combinatorics to convince people that they don't need to buy that many clothes?